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Discussion

Response to "Comment on paper 'The bulk modulus and Poisson's ratio of "incompressible" materials'"

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ABSTRACT

The incorrectness of the common assumption that rubbery polymers are incompressible does not preclude its yielding accurate determinations of the elastic modulus for nonlinear deformations.

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Voinovich [1] makes two points:

(i) "There is neither physical nor mathematical reason for the bulk modulus to tend to infinity". This reiterates statements in our paper [2] and thus there is no disagreement.

(ii) "The Young modulus E and the shear modulus G become zero when $\nu = \frac{1}{2}$". This is correct but pedantic. Rubbers are often described as being subjected to "incompressible deformation", since the bulk modulus B is on the order of 2000G, so that for practical purposes there is no volume change when the material undergoes appreciable elastic deformation. Moreover, in the development of nonlinear elastic constitutive theories of rubber (for a review see Ref. [3]), the pressure term of the stress tensor is not considered. This is a useful approximation, analogous to "incompressible flow" in fluid mechanics. As an elastomer approaches the softening zone ($G/B \rightarrow 0$) and conforms to "incompressible" rubber elasticity, an unfortunate misinterpretation of some workers is that the bulk modulus becomes very large. The purpose of [2] was to clarify this issue.

Notwithstanding, it is misleading to adopt the view of [1] that the assumption of incompressibility *requires* a zero shear modulus. Finite element modeling of elastomer products such as tires commonly assumes that $\nu = \frac{1}{2}$; indeed, the default value of the bulk modulus in commercial modeling software for rubber is usually infinity [4,5,6]. Nevertheless, these programs can yield accurate estimates of tensile and shear moduli.

Eq. (2) of [1] is a restatement of Zeno's Arrow Paradox [7] and only tangentially relevant.

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